As you can see from Derivations 35.1 and 35.2, the phenomena of time dilation and length contraction are quite intimately related. The speed of light being constant for all observers implies time dilation, which has been experimentally confirmed many times. Time dilation, in turn, implies length contraction.

The essential fact to remember from our deliberations on length contraction is that moving objects are shorter. They don't just appear shorter—they *are* shorter as measured by an observer in the frame in which the subject is moving. This contraction is another mind-bending consequence of the postulates of special relativity.

EXAMPLE 35.3 Length Contraction of a NASCAR Race Car

You see a NASCAR race car (Figure 35.8) go by at a constant speed of v = 89.4 m/s (200 mph). When stopped in the pits, the race car has a length of 5.232 m.

PROBLEM

What is the change in length of the NASCAR race car from your reference frame in the grandstands? Assume the car is moving perpendicular to your line of sight.

SOLUTION

The length of the race car will be contracted because of its motion. The proper length of the race car is L_0 = 5.232 m. The length in our reference frame is given by equation 35.10:

$$L = \frac{L_0}{\gamma} = L_0\sqrt{1-(v/c)^2} \approx L_0\left(1-\frac{1}{2}\left(\frac{v}{c}\right)^2\right) = L_0 - \Delta L,$$

where

$$\Delta L = L_0\frac{1}{2}\left(\frac{v}{c}\right)^2$$

is the change in length of the race car. Here we have applied a series expansion $(1-x^2)^{1/2} = 1-\frac{1}{2}x^2+\cdots$ as we did in equation 35.4. The car's speed is small compared with the speed of light, so $v/c \ll 1$ and our expansion is well justified. Thus, the race car appears to be shorter by

$$\Delta L = L_0\frac{1}{2}\left(\frac{v}{c}\right)^2 = \frac{5.232\text{ m}}{2}\left(\frac{89.4\text{ m/s}}{3.00\cdot 10^8\text{ m/s}}\right)^2 = 2.32\cdot 10^{-13}\text{ m}.$$

The car's change in length is smaller than the diameter of a typical atom. So the length contraction of objects at everyday speeds is not easy to observe.

Twin Paradox

We have seen that a time interval (say, between clock ticks) depends on the speed of the object (say, a clock) in the frame of an observer, $\Delta t = \gamma\Delta t_0$. Let's perform a little thought experiment:

Astronaut Alice has a twin brother, Bob. At the age of 20, Alice boards a spaceship that flies to a space station 3.25 light-years away and then returns. The spaceship is a good one and can fly with a speed of 65.0% of the speed of light, resulting in a gamma factor of $\gamma = 1.32$. The total distance traveled by Alice is 2(3.25 light-years) = 6.50 light-years as seen by Bob.

In Alice's rest frame, she travels a distance of d = 6.50 light-years/γ = 4.92 light-years, because the distance between Earth and the space station is length-contracted in her reference frame. Thus, the time it takes Alice to complete the trip is

$$t = d/v = (4.92c\cdot\text{years})/0.650c = 7.57\text{ years}.$$

35.4 In-Class Exercise

State whether each of the following statements is true or false.

a) For a moving object, its length along the direction of motion is shorter than when it is at rest.

b) When you are stationary, a clock moving past you at a significant fraction of the speed of light seems to run faster than the watch on your wrist.

c) When you are moving with a speed that is a significant fraction of the speed of light, and you pass by a stationary observer, you observe that your watch seems to be running faster than the watch of the stationary observer.

FIGURE 35.8 A NASCAR race car.

While the entire trip back and forth takes Alice 7.6 years in Alice's reference frame, time dilation forces $7.6\gamma = 10.0$ years to pass in Bob's reference frame. Therefore, when Alice steps out of the spaceship after her trip, she will be 27.6 years old, whereas Bob is 30.0 years old.

Now we can also put ourselves into the reference frame of Alice: In Alice's frame she was at rest and Bob was moving at 65.0% of the speed of light. Therefore, Alice should have aged 1.32 times more than Bob aged. Because Alice knows that she has aged 7.57 years, she might expect her brother Bob to be only $(20 + 7.57/1.32 = 25.8)$ years old when they meet again. Both siblings cannot each be younger than the other. This apparent inconsistency is called the twin paradox. Which of these two views is right?

The apparent paradox is resolved when we realize that although Bob remains in an inertial reference frame at rest on the Earth for the duration, astronaut Alice lives in two different inertial frames during her round trip. During the outbound leg, she is moving away from Earth and toward the distant space station. When she reaches the space station, she turns around and travels back from the space station at a constant speed to Earth. Thus, the symmetry is broken between the two twins.

We can analyze the path of the two twins in space-time by using our techniques of light cones and world lines, plotting time in an inertial rest frame versus the position of both twins in one direction, the x-direction. We analyze the problem from both the point of view of Bob and the point of view of Alice. We start by analyzing the trip in the rest frame of stay-at-home twin Bob, as shown in Figure 35.9. Here we scale the axes so that the units for both are in years.

In Figure 35.9, Bob's speed is always zero and he remains at $x = 0$. A red, vertical line represents Bob's trajectory. In contrast, Alice is moving with a speed of 65.0% of the speed of light ($v = 0.650c$) away from Earth. A blue line labeled $v = 0.650c$ depicts Alice's outbound trajectory. We define the positive x-direction as pointing from the Earth to the distant space station. Each twin wants to keep in touch with the other. Thus, each twin sends an electronic birthday card to the other twin on their birthday in their reference frame. These messages travel with the speed of light. Bob sends his electronic message directly toward the space station and Alice sends her electronic greeting back directly toward Earth. Bob's messages are shown as red arrows pointing up and to the right. Alice's messages are shown as blue arrows pointing up and to the left. When the message arrows cross the trajectory of each of the twins, the respective twin receives and enjoys the electronic birthday card.

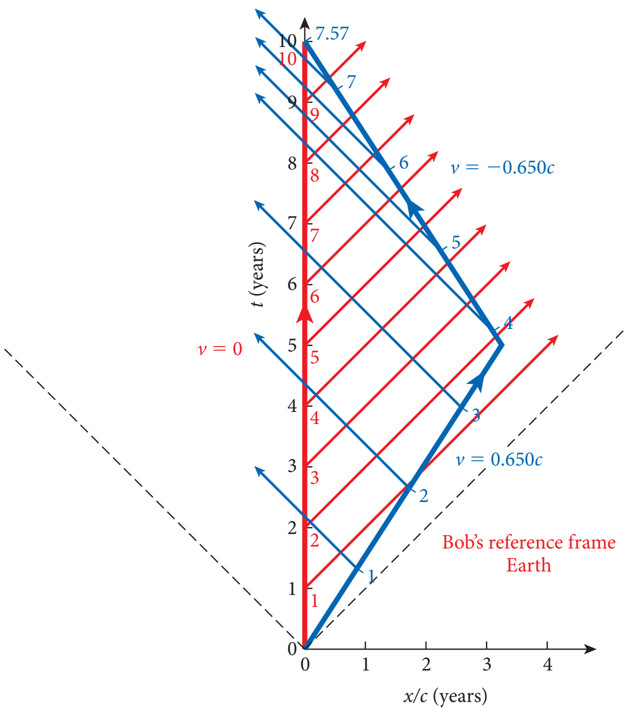

FIGURE 35.9 Plot showing the velocity of the two twins in Bob's reference frame, which is at rest on Earth. The thick, vertical red line represents Bob's trajectory. The two thick blue lines depict Alice's trajectory. Thin red lines labeled by red numbers (corresponding to the years since Alice left) represent Bob's birthday messages. Thin blue lines labeled by blue numbers (also corresponding to the years since Alice left) depict Alice's birthday messages. The dashed lines show the light cone at $t = 0$.

After 5 years pass in Bob's frame and 3.79 years pass in Alice's frame, Alice reaches the space station and turns for home. Bob is getting a little worried by now because he has received only two birthday cards in five years. Alice is not feeling much better, since she has received only one message in 3.79 years. After Alice turns around, she receives eight electronic birthday cards in the next 3.79 years. Bob receives five more greetings in the remaining five years. When Alice arrives back home on Earth, she gets a firsthand 30th birthday greeting from Bob, but Alice is not ready to celebrate her 28th birthday yet. Alice's age is 27.6 years. Alice received a total of ten birthday greetings while Bob received only seven.

Now let's analyze the same trip from the inertial rest frame corresponding to Alice's outward-bound leg (Figure 35.10) to show that we can get the same answer from Alice's point of view. In this reference frame, Bob and Earth are both traveling in the negative x-direction with a velocity $v = -0.650c$. During the outbound portion of the trip, Alice's velocity is zero in this frame. The space station is traveling toward Alice with a speed of 65.0% of the speed of light, so the distance of 3.25 light-years is covered in 3.79 years due to length contraction. Note that this part of the two diagrams in Figure 35.9 and Figure 35.10 is completely symmetric.

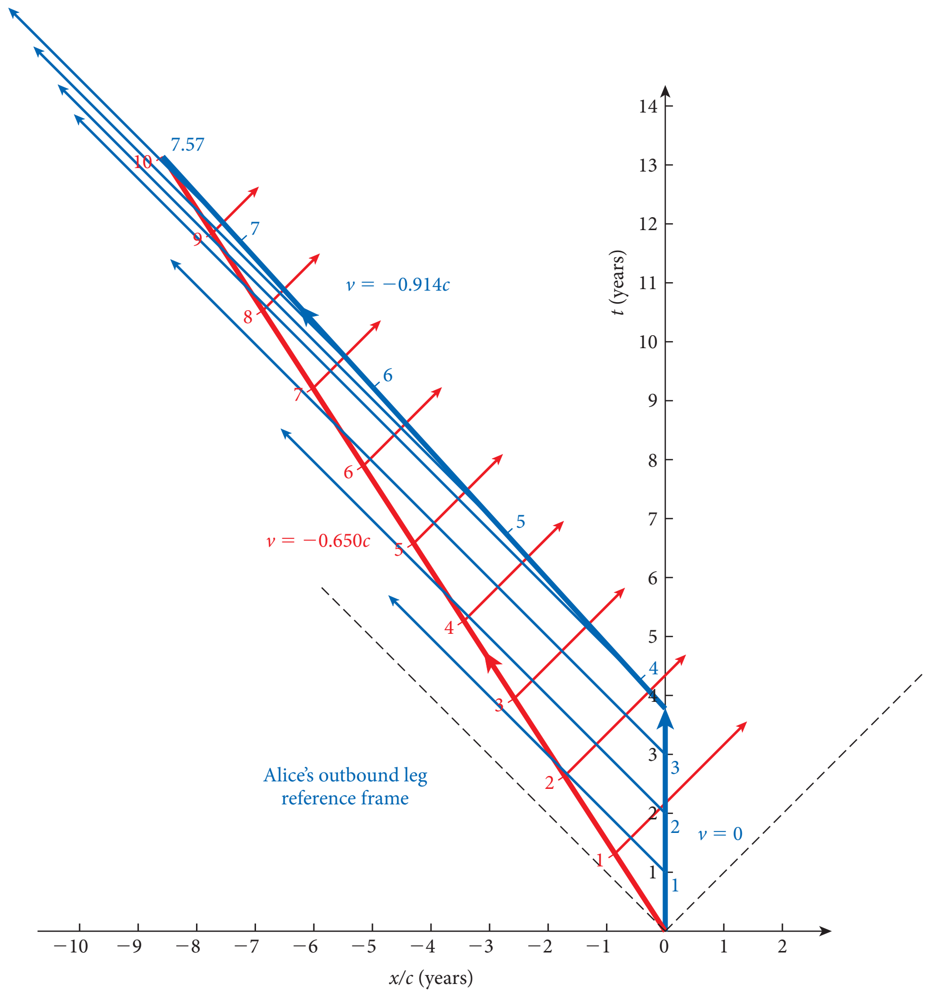

FIGURE 35.10 Plot showing the velocity of the two twins in the reference frame of Alice's outbound leg. The thick red line represents Bob's trajectory. The two thick blue lines depict Alice's trajectory. Thin red lines labeled by red numbers corresponding to the year represent Bob's birthday messages. Thin blue lines labeled by blue numbers corresponding to the year depict Alice's birthday messages. The dashed lines represent the light cone at $t = 0$.

When the space station reaches Alice, the symmetry in the two representations of Figures 35.9 and 35.10 is broken. Alice begins to travel with a speed fast enough to catch up with Earth in the negative x-direction. To establish a relative speed of 65.0% of the speed of light with respect to Earth, Alice must travel with a speed of 91.4% of the speed of light. (This relativistic addition of velocities is discussed in Section 35.6.)

Again Bob receives two birthday greetings in the first five years in his reference frame, while Alice again receives one birthday greeting before she starts moving toward Earth. As Alice streaks for Earth at a speed of $0.914c$, she receives eight birthday greetings, while Bob again receives five. When the twins are reunited on Earth, Bob is 30.0 years old and Alice is 27.6 years old. This result using Alice's outbound reference frame is the same as the one we obtained using Bob's reference frame. Thus, the twin paradox is resolved.

Note that we analyzed the twin paradox in terms of special relativity only. You might worry about the parts of Alice's journey that involved acceleration. Alice had to accelerate to 65.0% of the speed of light to begin her journey to the space station, and then she had to slow down, stop, and reaccelerate back up to a speed of 65.0% of the speed of light back toward Earth. Even at a constant acceleration of three times the acceleration of gravity, it would take almost three months to reach a speed of 65.0% of the speed of light from rest and the same amount of time to stop, starting at a speed of 65.0% of the speed of light. However, we could postulate various scenarios to remove these objections or at least minimize their importance. For example, we could simply make the trip longer, making the acceleration phase negligible. Acceleration is necessary to explain the twin paradox because Alice must reverse her course to return to Earth, changing her inertial reference frame. However, the effects of general relativity are not needed to explain the twin paradox.

35.5 In-Class Exercise

The nearest star to us other than the Sun is Proxima Centauri, which is 4.22 light-years away. Suppose we had a spaceship that could travel at a speed of 90.0% of the speed of light. If you were in the spaceship, how long would it take for you to travel from the Sun to Proxima Centauri, from your point of view?

a) 2.04 years
b) 2.29 years
c) 3.42 years
d) 3.80 years
e) 4.22 years